DBX-003-0492002   Seat No. _____

B. Sc./M. Sc. (Applied Physics) (CBCS) (Sem. II) Examination

July - 2022

Paper - VI : Applied Mathematics

(New Course)

Faculty Code : 003
Subject Code : 0492002

Time : $2\frac{1}{2}$ Hours]   [Total Marks : **70**

Instructions : (1) All questions are compulsory.
                    (2) Numbers in the right margin indicate marks.

1   Attempt any **SEVEN** short questions : (Two marks each) **14**

   (1) Find the degree of the homogeneous function $z = \dfrac{1}{y} + \dfrac{1}{x}$.

   (2) Find the *I.F.* of the differential equations $y' = x + y$.

   (3) Find the total differential of $z = xy$.

   (4) Find z_x and z_y for the surface $z = xy + z^2$

   (5) Form the partial differential equation from $z = f(x^2+y^2)$.

   (6) Solve : $t = z$.

   (7) Interpret geometrically Langrange' theorem.

   (8) Evaluate $\int_0^1 \int_{-1}^1 xy\, dxdy$

2   (A) Write answers of any **TWO** : (Two marks each)   **04**

   (1) Solve : $e^x dy - (ye^x + 2x)dx = 0$

   (2) Do test for exactness : $(x^2 - ay)dx + (y^2 - ax)dy = 0$

   (3) Solve : $\dfrac{dy}{dx} + \left(\dfrac{y}{x}\right) = 2$

   (4) Solve : $y = px - p^2 + \cos p = 0$

2   **(B)**   Write answers of any **TWO** : (**Five** marks each)   **10**

     (1)   Solve : $y' = \dfrac{y}{x} + \sin\dfrac{y}{x}$

     (2)   Solve : $e^{-y}\sec^2 y\, dy = dx + x\, dy$

     (3)   Solve : $y^2 P^2 - x^2 = 0$

     (4)   Solve : $xp - y + \sqrt{x} = 0$

3   **(A)**   Write answer of any **TWO** : (**Two** marks each)   **04**

     (1)   If $z = y^x$ then find z_x and z_y.

     (2)   If $x = u(1{-}v)$ and $y = uv$ then find $J\left(\dfrac{x,y}{r,\theta}\right)$.

     (3)   If $u = \sin\dfrac{x}{y} + \sin\dfrac{y}{x}$, then prove that $xu_x + yu_y = 0$.

     (4)   If $u = e^x(x\cos y - y\sin y)$, then prove that $u_{xx} + u_{yy} = 0$.

3   **(B)**   Write answer of any **TWO** : (**Five** marks each)   **10**

     (1)   If $\sin u = \dfrac{x^2 y^2}{x+y}$ then show that $xu_x + yu_y = 3\tan u$

     (2)   If $u = \tan^{-1}\dfrac{x^2 y^2}{x+y}$ then prove that $xu_x + yu_y = \sin 2u$.

     (3)   Find the extremum value of

$$f(x,y) = x^2 - 2xy - \frac{y^3}{3} - 3y.$$

     (4)   Find the area of greatest rectangle that can be inscribed in the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ (hint : area function f(x,y)=xy)

4   **(A)**   Write answers of any **TWO** : (**Two** marks each)   **04**

     (1)   Form the partial differential equation for
        $z = ax + by + ab$.

     (2)   Solve : $\dfrac{d^2 u}{dx\, dt} = e^{-t}\cos x$

     (3)   Solve : $p + q = qz$

     (4)   Solve : $p + q = x + y$.

4 (B) Write answers of any **TWO** : (**Five** marks each) 10

 (1) Solve : $pyz = qzx = xy$.

 (2) Solve : $z^2 (p^2x^2 + q^2)=1$

 (3) Solve : $2r + 5s + 2t= e^{x+y}$.

 (4) Solve : $py^3 + qx^2 = 0$

5 (A) Write answers of any **TWO** : (**Two** marks each) 04

 (1) Interpret Roll's theorem geometrically.

 (2) Verify Roll's theorem for $f(x) = x^2 - 2x, \forall\, x \in [-1, 3]$.

 (3) Write the infinite expansion of e^x. Deduce the infinite series for e. (Hint $x =1$)

 (4) Derive the expansion of $e^x sinx$.

5 (B) Write answers of any **TWO** : (**Five** marks each) 10

 (1) Show that $\dfrac{x}{1+x} < \log(1+x) < x$ for $x > 0$

 (2) Expand $f(x) = x^4 - 11x^3 + 43x^2 - 60x + 14$ in ascending power of $(x - 3)$.

 (3) Evaluate $\iint_R xydxdy$ where R is first quadrant of the circle $x^2 + y^2 = a^2$

 (4) Find the volume of the unit ball.

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